Instructions: Complete each of the following as practice.

1. Express each of the following linear systems as a matrix equation. Classify each system as homogeneous or non-homogeneous.

(a) $\begin{cases} x + y + z = 3 \\ 3x - 5y + z = 0 \\ y + z = 2 \end{cases}$

(b) $\begin{cases} x_1 + x_2 + 4x_3 + 3x_4 = 5 \\ 2x_1 + 3x_2 + x_3 - 2x_4 = 1 \\ x_1 + 2x_2 - 5x_3 + 4x_4 = 3 \end{cases}$

(c) $\begin{cases} x + y - z = 0 \\ 2x - 3y + z = 0 \\ x - 4y + 2z = 0 \end{cases}$

(d) $\begin{cases} 4x - 6y = 8 \\ -6x + 9y = 6 \end{cases}$

(e) $\begin{cases} x + y - z = 0 \\ 2x + 4y - z = 0 \\ 3x + 2y + 2z = 0 \end{cases}$

(f) $\begin{cases} x + 2y - 4z = -4 \\ 2x + 5y - 9z = -10 \\ 3x - 2y + 3z = 11 \end{cases}$

(g) $\begin{cases} x + 2y - 3z = -1 \\ -3x + y - 2z = -7 \\ 5x + 3y - 4z = 2 \end{cases}$

(h) $\begin{cases} x + 3y - 3z = 1 \\ 2x + 5y - 8z = 4 \\ 3x + 8y - 13z = 7 \end{cases}$

2. For each linear system from question 1, state and solve the associated homogeneous linear system. *Give the complete solution set as a set of column vectors.*

3. State the size of each matrix given below.

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 6 & 0 \\ 3 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 0 & 2 \\ 3 & 2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \quad E = \begin{bmatrix} 2 & 6 \\ -1 & 2 \\ 3 & 1 \\ -1 & 3 \end{bmatrix} \quad F = \begin{bmatrix} 2 & -1 & 3 & -1 \\ 6 & 2 & 1 & 3 \end{bmatrix}$$

4. Let A, B, C, D, E and F be the matrices given in question 3. Which of the following matrix operations are well-defined? Compute the results where meaningful and explain why the expression is undefined otherwise.

(a) $A + B$
(b) $A - 3B$
(c) AB
(d) $3BA$
(e) AA
(f) AC
(g) CA
(h) AD
(i) $2DA$
(j) AE
(k) $-2EA$
(l) $3E$
(m) $A + 2C$
(n) $D + C$
(o) CD
(p) DC
(q) $E + F$
(r) FE
(s) EF
(t) $(DA)C$

5. For further exercises, see the following (note: this list may break with future versions of these textbooks).

(a) Beezer page 172 (problems C10 – C14)
(b) Hefferon page 20 (problems 2.15 – 2.30), page 33 (problems 3.14 – 3.18), page 241 (problems 2.14 – 2.18)
(c) Matthews chapter 2.1